

泊松脉冲激励的多自由度耗散哈密顿系统平稳响应

吴 禹, 朱位秋

(浙江大学航空航天学院应用力学研究所, 杭州 310027)

摘要: 本文研究了随机脉冲激励下的多自由度耗散哈密顿系统的平稳概率与统计特性。随机脉冲采用 Poisson 白噪声模型。通过用摄动法求解四阶广义 Fokker-Planck 方程, 得到 Poisson 白噪声激励的多自由度耗散哈密顿系统的近似平稳响应概率密度函数和均方值。作为例子, 考察了两自由度线性与非线性刚度耦合的 Duffing 振子, 证明了其响应的非高斯特性依赖于脉冲平均到达率和系统弛豫时间的乘积, 并用 Monte Carlo 模拟验证了该方法的可行性。

关键词: 工程力学; 泊松白噪声; 摄动法; 耗散的哈密顿系统; 概率密度函数; 广义 Fokker-Planck 方程

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Stationary response of multi-degree-of-freedom dissipated Hamiltonian systems under Poisson pulses

WU Yu, ZHU Weiqiu

(Institute of Applied Mechanics, School of Aeronautics and Astronautics,
Zhejiang University, Hangzhou 310027)

Abstract: The paper studied the stationary probabilistic and statistical characteristics of multi-degree-of-freedom (MDOF) dissipated Hamiltonian systems driven by random pulses. The pulse model adopted is Poisson white noise. The approximate stationary probability density functions and mean square values for the response of MDOF dissipated Hamiltonian systems to Poisson white noise are obtained by solving the four-order generalized Fokker-Planck equation using perturbation approach. As an example, two Duffing oscillators coupled with linear and nonlinear stiffness terms are investigated. It is shown that the non-Gaussian behavior depends on the product of the average arrival rate of the impulses and the relaxation time of the oscillator. The validity of this approach is confirmed by experimental results obtained by Monte Carlo simulation.

Key words: engineering mechanics; Poisson white noise; perturbation approach; dissipated Hamiltonian system; probability density function; generalized Fokker-Planck equation

0 引言

随机脉冲序列是一大类工程中的随机振源, 包括地震^[1~2]、车辆对桥梁的载荷^[3]、铁轨与车轮之间的冲击^[3~4]、机械结构的碰撞^[5]、风和波浪对船体或海洋平台等离岸结构的冲击^[6~7]等的有效模型。它属于非高斯随机过程, 相比常见的高斯白噪声模型, 更能反映上述激励的真实特性, 所以对这类非高斯随机脉冲序列的研究具有重要的现实意义。

近二十年来, 在非线性系统对非高斯随机脉冲序列的响应领域进行了大量研究。高斯白噪声激励情形的 Itô 微分规则已被推广至泊松白噪声激励情形^[8], 并推导了泊松白噪声激励下非线性系统随机微分方程的 Stratonovich 修正项和广义 Fokker-Planck 方程^[9], 由于广义 Fokker-Planck 方程包含无穷多项, 难以精确求解, 目前仅得到一些极为特殊情况下—维系统的精确平稳解^[10~11], 对二维以上情形

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作者简介: 吴禹 (1980—), 男, 博士研究生, 主要研究方向: 非高斯随机激励下非线性系统的响应与可靠性

通信联系人: 朱位秋, 教授, 主要研究方向: 非线性随机动力学与控制。E-mail: wqzhu@yahoo.com

一般只能考虑近似解。对无穷阶 Fokker-Planck 方程常采取截断的办法,除高斯情形的前两阶项外,仅保留分别反映非高斯概率密度的两个主要特征(偏度和峰度)的三阶和四阶项^[12]。迄今对该领域的研究还基本局限于线性系统或单自由度非线性系统。ROBERTS^[13]最先提出用摄动法求解 Fokker-Planck 方程并应用于线性系统,CAI 等^[14]将该方法改进并推广到单自由度非线性系统。TYLIKOWSKI 等^[15]和 GRIGORIU^[16~17]将等效线性方法应用于泊松白噪声和列维白噪声激励的单自由度非线性系统。IWANKIEWICZ 等^[18~19]和 KÖYLÜOĞLU 等^[20~21]发展了随机脉冲激励的非线性系统的矩方程截断近似法、Peterov-Galerkin 法和胞映射法。GRIGORIU^[22]提出了一种求解响应特征函数所满足的偏微分方程的方法。

本文将摄动法应用于求多自由度耗散哈密顿系统对随机脉冲激励的平稳响应,以高斯白噪声激励下的耗散哈密顿系统的精确平稳概率密度^[23]作为零级摄动解,通过对无量纲小量的参数摄动得到非高斯激励情形的近近平稳概率密度函数的修正项。作为例子,该方法应用于一个泊松白噪声外激的线性和非线性刚度耦合的两自由度 Duffing 振子,并用 Monte Carlo 模拟验证了其可行性。

1 泊松白噪声脉冲模型

脉冲序列模型已经得到越来越多的关注。一种常用的脉冲序列模型是泊松白噪声,这是对实际脉冲激励的一种较好的近似,描述了在随机时刻到达的幅值满足特定分布的脉冲序列

$$\xi(t) = \sum_{k=1}^{N(t)} Y_k \delta(t - t_k) \quad (1)$$

式中, $N(t)$ 为泊松计数过程; 脉冲幅值 Y_k 为满足一定分布的相互独立的随机变量,并且也独立于脉冲到达时刻 t_k ; 泊松白噪声的相关函数为^[8,24]

$$R^{(r)}[\xi(t_1), \xi(t_2), \dots, \xi(t_r)] = \lambda E[Y^r] \delta(t_2 - t_1) \cdots \delta(t_r - t_1), r = 1, 2, \dots, \infty \quad (2)$$

式中, $\lambda > 0$ 是泊松计数过程的平均到达率,当 $\lambda \rightarrow \infty$ 且 $\lambda E[Y^2]$ 保持常值时,泊松白噪声趋近高斯白噪声。

泊松白噪声可视为如下齐次复合泊松过程的形式导数

$$C(t) = \sum_{k=1}^{N(t)} Y_k U(t - t_k) \quad (3)$$

易证其增量 $dC(t_i) = C(t_i + dt) - C(t_i)$ 的相关函数为

$$R^{(r)}[dC(t_1), dC(t_2), \dots, dC(t_r)] = \lambda E[Y^r] \delta(t_2 - t_1) \cdots \delta(t_r - t_1) dt_1 dt_2 \cdots dt_r \quad (4)$$

这显示复合泊松过程是独立增量的过程。其增量的各阶矩满足以下关系^[8]

$$E[(dC(t))^r] = \lambda E[Y^r] dt \quad (5)$$

2 多自由度广义 Fokker-Planck 方程

考虑泊松白噪声激励下的耗散哈密顿系统^[24]

$$\begin{aligned} dQ_i &= \frac{\partial H}{\partial P_i} dt \\ dP_i &= \left(-\frac{\partial H}{\partial Q_i} - c_{ij}(Q, P) \frac{\partial H}{\partial P_j} + \frac{\lambda}{m!} E[Y_{k_1} Y_{k_2} \cdots Y_{k_m}] S_{k_1 k_2 \cdots k_m}^{(m)} \right) dt + f_{ik}(Q, P) dC_k(t) \end{aligned} \quad (6)$$

($i, j = 1, 2, \dots, n; k, l = 1, 2, \dots, \alpha$)

式中, Q, P 分别代表广义位移和广义动量的随机过程, $\lambda/m! E[Y_{k_1} Y_{k_2} \cdots Y_{k_m}] S_{k_1 k_2 \cdots k_m}^{(m)}$ 为 Wong-Zakai-Paola-Falsone (WZPF) 修正项, 可通过如下迭代得到

$$S_{k_1 k_2 \dots k_m}^{(m)} = \frac{\partial S_{k_1 k_2 \dots k_{m-1}}^{(m-1)}}{\partial p_j} f_{jk_m}(\mathbf{Q}, \mathbf{P}), S_{k_1}^{(1)} = f_{k_1}(\mathbf{Q}, \mathbf{P}) \quad (7)$$

当 f_{ij} 仅依赖于 $\mathbf{Q}$ 时, WZPF 修正项消失, 平稳概率密度函数 $p(\mathbf{q}, \mathbf{p})$, $\mathbf{q}, \mathbf{p}$ 分别为广义位移和广义动量的随机变量, 满足如下齐次广义 FPK 方程

$$\begin{aligned} & -\frac{\partial}{\partial q_i} \left(\frac{\partial H}{\partial p_i} p \right) - \frac{\partial}{\partial p_i} \left[\left(-\frac{\partial H}{\partial q_i} - c_{ij} \frac{\partial H}{\partial p_j} \right) p \right] + \\ & \frac{1}{2} \delta_{mn} \lambda E[Y_m Y_n] f_{im} f_{in} \frac{\partial^2 p}{\partial p_i \partial p_j} - \frac{1}{3!} \delta_{mn} \delta_{mp} \lambda E[Y_m Y_n Y_p] f_{im} f_{in} f_{kp} \frac{\partial^3 p}{\partial p_i \partial p_j \partial p_k} + \\ & \frac{1}{4!} \delta_{mn} \delta_{mp} \delta_{nq} \lambda E[Y_m Y_n Y_p Y_q] f_{im} f_{in} f_{kp} f_{lq} \frac{\partial^4 p}{\partial p_i \partial p_j \partial p_k \partial p_l} - \dots \\ & = 0 \quad (i, j, k, l = 1, 2, \dots, n; m, n, p, q = 1, 2, \dots, \alpha) \end{aligned} \quad (8)$$

3 摄动法

假设泊松脉冲的 α 个独立分量的平均到达率 λ 和强度分别相等, 即

$$\lambda_i = \lambda, \quad \lambda_i E[Y_i^2] = \lambda E[Y^2], \quad i = 1, 2, \dots, \alpha \quad (9)$$

则有

$$D_{nn} = \frac{1}{2} \lambda E[Y^2] \equiv D_2, \quad D_{nnp} = \frac{1}{2} \lambda E[Y^3] \equiv D_3, \quad D_{nmpq} = \frac{1}{2} \lambda E[Y^4] \equiv D_4 \quad (10)$$

由于脉冲激励强度有限, $\lambda E[Y^2]$ 也必有限。选择摄动小参数 $\epsilon = (\lambda/\zeta)^{-1/2}$, 使得 $\lambda E[Y^2] \sim O(1)$, 则有: $\zeta E[Y^2] \sim O(\epsilon^2)$, $\zeta^{3/2} E[Y^3] \sim O(\epsilon^3)$, $\zeta^2 E[Y^4] \sim O(\epsilon^4)$, 当 $\lambda/\zeta \rightarrow \infty$ 时, $\epsilon \rightarrow 0$, 泊松白噪声激励趋向高斯白噪声激励。

令

$$\epsilon^n I_n = \zeta^{n/2} \lambda E[Y^{n+2}] \quad (11)$$

寻求如下形式摄动解

$$p(\mathbf{q}, \mathbf{p}) = p_0(\mathbf{q}, \mathbf{p}) [1 + \epsilon Q_1(\mathbf{q}, \mathbf{p}) + \epsilon^2 Q_2(\mathbf{q}, \mathbf{p}) + \dots] \quad (12)$$

式 (12) 代入式 (8), 按 ϵ 幂次分组, 得如下二阶偏微分方程组

$$\epsilon^0: -\frac{\partial}{\partial q_i} \left(\frac{\partial H}{\partial p_i} p_0 \right) + \frac{\partial}{\partial p_i} \left[\left(\frac{\partial H}{\partial q_i} + c_{ij} \frac{\partial H}{\partial p_j} \right) p_0 \right] + \frac{I_0}{2} \delta_{mn} f_{im} f_{in} \frac{\partial^2 p_0}{\partial p_i \partial p_j} = 0 \quad (13)$$

$$\begin{aligned} \epsilon^1: & -\frac{\partial H}{\partial p_i} \frac{\partial Q_1}{\partial q_i} + \left(\frac{\partial H}{\partial q_i} + c_{ij} \frac{\partial H}{\partial p_j} + 2I_0 \delta_{mn} f_{im} \frac{\partial f_{in}}{\partial p_j} + \frac{I_0}{p_0} \delta_{mn} f_{im} f_{in} \frac{\partial p_0}{\partial p_j} \right) \frac{\partial Q_1}{\partial p_i} \\ & + \frac{I_0}{2} \delta_{mn} f_{im} f_{in} \frac{\partial^2 Q_1}{\partial p_i \partial p_j} = \frac{I_1}{3! \zeta^{1/2} p_0} \delta_{mn} \delta_{mp} f_{im} f_{in} f_{kp} \frac{\partial^3 p_0}{\partial p_i \partial p_j \partial p_k} \end{aligned} \quad (14)$$

$$\begin{aligned} \epsilon^2: & -\frac{\partial H}{\partial p_i} \frac{\partial Q_2}{\partial q_i} + \left(\frac{\partial H}{\partial q_i} + c_{ij} \frac{\partial H}{\partial p_j} + 2I_0 \delta_{mn} f_{im} \frac{\partial f_{in}}{\partial p_j} + \frac{I_0}{p_0} \delta_{mn} f_{im} f_{in} \frac{\partial p_0}{\partial p_j} \right) \frac{\partial Q_2}{\partial p_i} + \frac{I_0}{2} \delta_{mn} f_{im} f_{in} \frac{\partial^2 Q_2}{\partial p_i \partial p_j} \\ & = \frac{I_1}{3! \zeta^{1/2} p_0} \delta_{mn} \delta_{mp} f_{im} f_{in} f_{kp} \frac{\partial^3 p_1}{\partial p_i \partial p_j \partial p_k} - \frac{I_2}{4! \zeta p_0} \delta_{mn} \delta_{mp} \delta_{nq} f_{im} f_{in} f_{kp} f_{lq} \frac{\partial^4 p_0}{\partial p_i \partial p_j \partial p_k \partial p_l} \end{aligned} \quad (15)$$

$$\vdots$$

$$(i, j, k, l = 1, 2, \dots, n; m, n, p, q = 1, 2, \dots, \alpha)$$

零阶摄动式 (13) 正是高斯白噪声外激情形的 Fokker-Planck 方程, 对广义平稳势系统存在精确平稳解, 解的函数形式依赖于相应哈密顿系统的可积性与共振性^[23]。

借助式 (13) 的解 $p_0(\mathbf{q}, \mathbf{p})$ 可以得到式 (14) 的解。对任意的 H 和 f_{ij} , 式 (14) 极难求解。然而, 大多数情况下, H 是 q_i 和 p_i 的多项式, 如果同时 f_{ij} 是常数或 q_i 的线性函数, 则将 $\partial Q_i / \partial p_i$ 项的非线性系数线性化为 $A_i q_i + B_i p_i$, 可得到多项式形式的近似解。采用如下误差均方值最小作为线性化等效判据

$$E[\Delta^2] = \text{minimum} \quad (16)$$

式中

$$\Delta = A_i q_i + B_i p_i - \left(\frac{\partial H}{\partial q_i} + c_{ij} \frac{\partial H}{\partial p_j} + 2I_0 \delta_{mn} f_{in} \frac{\partial f_{im}}{\partial p_j} + \frac{I_0}{p_0} \delta_{mn} f_{im} f_{jn} \frac{\partial p_0}{\partial p_j} \right) \quad (17)$$

可得

$$\begin{aligned} A_i &= E \left[q_i \left(\frac{\partial H}{\partial q_i} + c_{ij} \frac{\partial H}{\partial p_j} + \frac{3I_0}{2} f_{jn} \frac{\partial f_{im}}{\partial p_j} + \frac{I_0}{p_0} \delta_{mn} f_{im} f_{jn} \frac{\partial p_0}{\partial p_j} \right) \right] (E[q_i^2])^{-1} \\ B_i &= E \left[p_i \left(\frac{\partial H}{\partial q_i} + c_{ij} \frac{\partial H}{\partial p_j} + \frac{3I_0}{2} f_{jn} \frac{\partial f_{im}}{\partial p_j} + \frac{I_0}{p_0} \delta_{mn} f_{im} f_{jn} \frac{\partial p_0}{\partial p_j} \right) \right] (E[q_i^2])^{-1} \end{aligned} \quad (18)$$

式 (15) 左端与式 (14) 相同, 故可用相同的办法近似求解。

为了估计线性化系数 A_i 和 B_i 的最优值, 可以用上述摄动法进行迭代。首先通过式 (13) 的精确解 $p_0(\mathbf{q}, \mathbf{p})$ 得到 A_i 和 B_i 的初始估计值, 接着解出如式 (12) 的 $p(\mathbf{q}, \mathbf{p})$ 并得到 A_i 和 B_i 的改进值。如此循环直至收敛。

4 算例

考虑两个无量纲化刚度耦合 Duffing 振子对泊松白噪声的响应, 其运动微分方程为

$$\begin{aligned} \ddot{X}_1 + 2\zeta_1 \dot{X}_1 + k_1 X_1 + aX_2 + b(X_1 - X_2)^3 &= \xi_1(t) \\ \ddot{X}_2 + 2\zeta_2 \dot{X}_2 + k_2 X_2 + aX_1 + b(X_2 - X_1)^3 &= \xi_2(t) \end{aligned} \quad (19)$$

其哈密顿形式的方程为^[23]

$$\begin{aligned} \dot{Q}_1 &= \frac{\partial H}{\partial P_1}, \quad \dot{P}_1 = -\frac{\partial H}{\partial Q_1} - 2\zeta_1 \frac{\partial H}{\partial P_1} + \xi_1(t) \\ \dot{Q}_2 &= \frac{\partial H}{\partial P_2}, \quad \dot{P}_2 = -\frac{\partial H}{\partial Q_2} - 2\zeta_2 \frac{\partial H}{\partial P_2} + \xi_2(t) \end{aligned} \quad (20)$$

式中, 哈密顿量为

$$H(q_1, q_2, p_1, p_2) = \frac{p_1^2}{2} + \frac{p_2^2}{2} + \frac{1}{2} k_1 q_1^2 + \frac{1}{2} k_2 q_2^2 + a q_1 q_2 + \frac{b}{4} (q_1 - q_2)^4 \quad (21)$$

$\xi_1(t)$, $\xi_2(t)$ 为泊松白噪声, 设 $\zeta_1 = \zeta_2 = \zeta$, $D_1 = D_2 = D$, 则式 (19) 存在精确平稳解^[23]

$$p_0(q_1, q_2, p_1, p_2) = C_0 \exp \left[-\frac{2\zeta H}{D} \right]_{H=H(q_1, q_2, p_1, p_2)} \quad (22)$$

下面按前述摄动法, 求本系统的近似响应概率密度函数。假设脉冲幅值满足高斯分布, 则 $I_1 = 0$, $Q_1 = 0$, 只需考虑二阶摄动。按照前述摄动法步骤, 可得如下系统平稳响应的近似概率密度函数

$$\begin{aligned}
 p(\mathbf{q}, \mathbf{p}) &= p_0(\mathbf{q}, \mathbf{p})[1 + \epsilon^2 Q_2(\mathbf{q}, \mathbf{p})] \\
 &= C_1 \exp \left\{ -\frac{4\zeta}{I_0} \left[\frac{p_1^2}{2} + \frac{p_2^2}{2} + \frac{1}{2}k_1 q_1^2 + \frac{1}{2}k_2 q_2^2 + a q_1 q_2 + \frac{b}{4}(q_1 - q_2)^4 \right] \right\} \\
 &\quad \left(1 + \frac{\zeta^2}{3\lambda I_0^2} \left\{ -18I_0(p_1^2 + p_2^2) + 8(AP_1^3 q_1 + BP_2^3 q_2) + 12(p_1^4 + p_2^4)\zeta + \right. \right. \\
 &\quad \left. \frac{A}{A + 3\zeta^2} [9A^2 q_1^4 \zeta - 3p_1(p_1^3 + 6I_0 q_1)\zeta + 2Aq_1(-4p_1^3 - 9I_0 q_1 + 9p_1^2 q_1 \zeta)] + \right. \\
 &\quad \left. \left. \frac{B}{B + 3\zeta^2} [9B^2 q_2^4 \zeta - 3p_2(p_2^3 + 6I_0 q_2)\zeta + 2Bq_2(-4p_2^3 - 9I_0 q_2 + 9p_2^2 q_2 \zeta)] \right\} \right)
 \end{aligned} \quad (23)$$

式中, C_1 为正则化常数, 线性化系数为

$$\begin{aligned}
 A &= k_1 + \frac{bE[q_1^4] + aE[q_1 q_2] - 3bE[q_1^3 q_2] + 3bE[q_1^2 q_2^2] - bE[q_1 q_2^3]}{E[q_1^2]} \\
 B &= k_2 + \frac{bE[q_2^4] + aE[q_1 q_2] - 3bE[q_1 q_2^3] + 3bE[q_1^2 q_2^2] - bE[q_1^3 q_2]}{E[q_2^2]}
 \end{aligned} \quad (24)$$

在数值计算中, 选择阻尼系数 $\zeta=0.04$, 刚度 $k_1=1$, $k_2=1.5$, $a=1$, 假设脉冲幅值满足均值为 0 的正态分布, 且强度 $I_0=0.03$ 。系统弛豫时间用对应的线性系统 ($b=0$) 来近似, $t_r=1/\zeta=25$ 。图 1 (a), (b) 分别是弱非线性系统 ($b=0.1$) 和强非线性系统 ($b=1$) 情况下的位移 q_1 的近似平稳响应概率密度。图 2 (a), (b) 则是相应情形下 q_2 的概率密度。

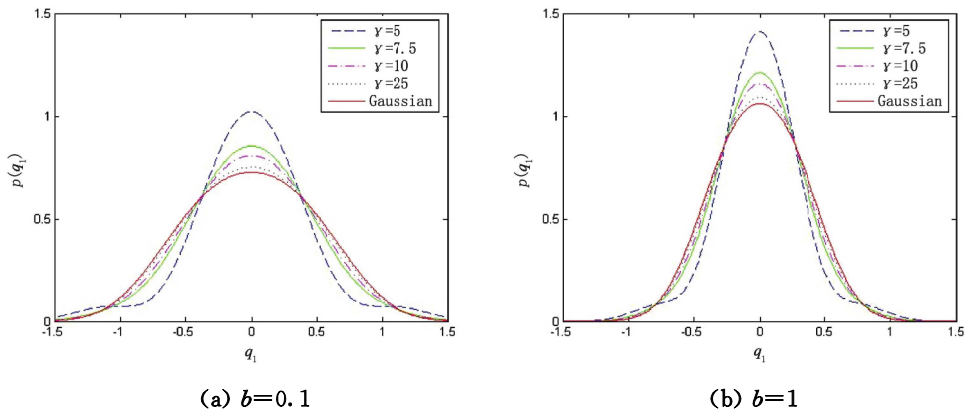


图 1 泊松白噪声激励的两自由度 Duffing 振子的平稳响应 q_1 的概率密度
Fig. 1 Approximate stationary probability density of displacement q_1 of 2DOF Duffing oscillator excited by additive Poisson white noise

脉冲的疏密对系统的响应将产生举足轻重的影响, 但脉冲是密集还是稀疏是一个相对的概念, 不能仅考虑脉冲的平均到达率, 同时还应考虑系统的固有特性, CAI 等^[14] 提出将脉冲平均到达率与系统弛豫时间的乘积作为判定标准, 即 $\gamma=\lambda t_r$ 。如图 1、图 2 所示, 列举了 $\gamma=5, 7.5, 10$ 和 25 四种情况, 并与高斯白噪声激励的情况比较。可见, γ 越小, 响应偏离高斯情形越显著, 当 $\gamma>25$ 时, 几乎可以忽略二者的差异。

图 3 (a), (b) 描绘了位移的近似平稳均方响应, 非线性系数分别为 $b=0, b=0.1$ 和 $b=1$ 。 $b=0$ 对应线性系统, 在所考察的范围内, 其均方响应不随 γ 而变, 并与线性系统响应理论中的结果一致^[14]。

为了验证本方法的可行性, 对本例的 Duffing 振子响应进行了 Monte Carlo 模拟, 结果如图 3 所示。可见, 在 $\gamma>5$ 的区域, 摄动法得到的结果与 Monte Carlo 模拟的结果较吻合。

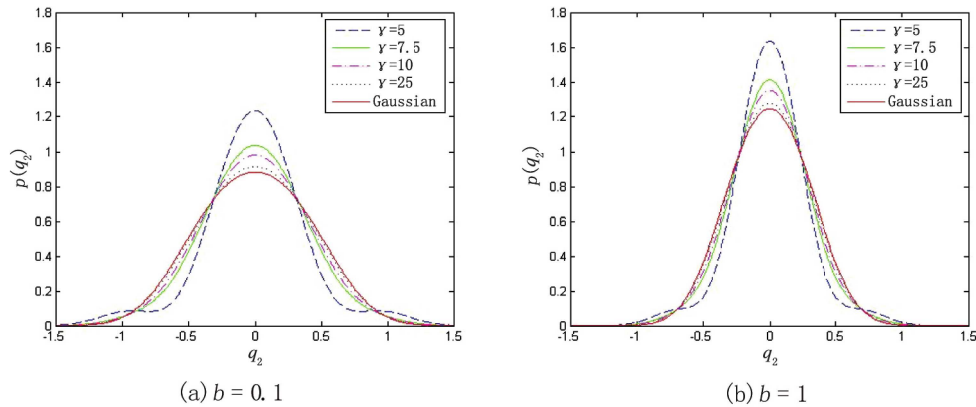


图 2 泊松白噪声激励的两自由度 Duffing 振子的平稳响应 q_2 的概率密度
Fig. 2 Approximate stationary probability density of displacement q_2 of 2DOF Duffing oscillator excited by additive Poisson white noise

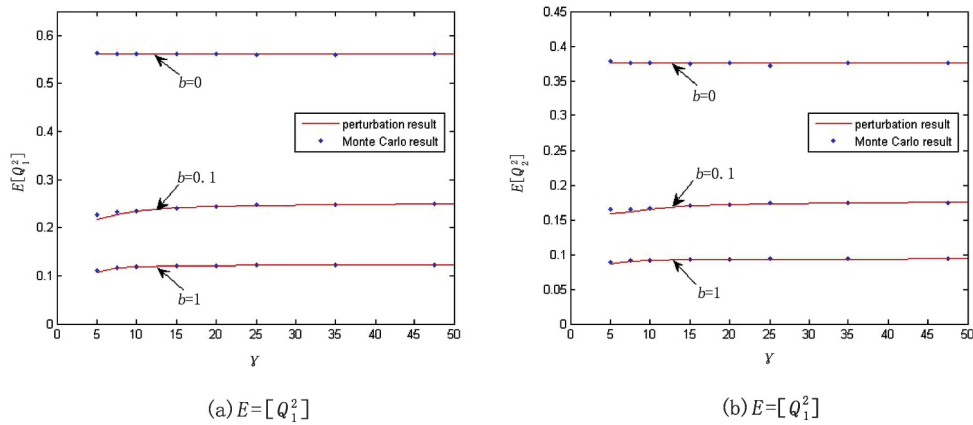


图 3 泊松白噪声激励的两自由度 Duffing 振子的平稳均方响应
Fig. 3 Mean square response of 2DOF Duffing oscillator excited by additive Poisson white noise

5 分析与讨论

本文采取 $\gamma = \lambda t_r$ ，而不是仅将 λ 作为非线性系统分别对泊松白噪声激励与高斯白噪声激励的响应偏离程度的度量标准，有着合理的物理解释，可从如下两个角度来看。

首先，从脉冲疏密程度的角度看，对平均到达率为 λ 的泊松白噪声而言，若采用简单的矩形脉冲模型或三角脉冲模型，其脉冲时间间隔的平均值近似为 $1/\lambda$ ，故系统弛豫时间与脉冲间隔时间的比值为 $\gamma = t_r/(1/\lambda) = \lambda t_r$ ，反映了脉冲对系统的“相对”密集度，这一比值越小，脉冲相对越稀疏。从文例 Duffing 振子的结果可见，当 $\gamma < 10$ 时，泊松白噪声激励与高斯白噪声激励二者的响应偏离较显著，而 $\gamma > 25$ 时，二者几乎没有差别。

其次，从单个脉冲强度的角度看， $\gamma = \lambda t_r = \lambda/\zeta = (I_0/\zeta)/E[Y^2]$ ，当噪声强度 I_0 一定时， $\gamma \propto (E[Y^2])^{-1}$ ， $E[Y^2]$ 表征了单个脉冲的强度（区别于噪声强度），其值越大，系统发生穿越损坏的概率也越大，因为阻尼系数为 ζ 的系统对单个脉冲的耗散能力是有限的。反映在平稳概率密度图上，0 点及其附近小值的概率变大的同时，出现较大响应的概率也变大（肥尾现象），整个概率峰变得陡峭。

6 结论

本文研究了随机脉冲激励的多自由度耗散哈密顿系统的平稳响应,选择泊松白噪声作为随机脉冲模型。通过摄动法求解广义 Fokker-Planck 方程,从而得到响应的近似平稳概率密度函数和均方值,并以一个泊松白噪声外激的两自由度刚度耦合 Duffing 振子作为例子,将得到的摄动结果与 Monte Carlo 模拟结果比较,证明了本方法的可行性。

结果表明,相对高斯白噪声激励的系统响应,在噪声强度一定的情况下,对泊松白噪声激励下的系统响应非高斯性态的强弱起决定性作用的是脉冲的相对疏密度。衡量相对疏密度的标准采用泊松计数过程的平均到达率与系统弛豫时间的乘积 $\gamma = \lambda t_r$, γ 越小,脉冲相对越稀疏,非高斯特性越显著。当 $\gamma > 25$ 时,泊松白噪声与高斯白噪声激励的系统响应几乎一致。尽管本文的算例只考虑了外激的情况,但本方法可以应用于参激多自由度非线性系统,随机脉冲模型也并不局限于泊松白噪声,可以发展为更一般的列维白噪声等。

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